Analysis of Variance

1. Randomized Block Design
2. Factorial Design

Analysis of Variance
Randomized Block Design

Randomized Block Design

1. Experimental Units (Subjects) Are Assigned Randomly within Blocks
   - Blocks are Assumed Homogeneous
2. One Factor or Independent Variable of Interest
   - 2 or More Treatment Levels or Classifications
3. One Blocking Factor

Randomized Block Design

Factor Levels: (Treatments)	A, B, C, D			
Experimental Units	Treatments are randomly assigned within blocks			
Block 1	A	C	D	B
Block 2	C	D	B	A
Block 3	B	A	D	C
. . .	. . .	. . .	. . .	. . .
Block b	D	C	A	B

Randomized Block F-Test

1. Tests the Equality of 2 or More (p) Population Means

2. Variables
   - One Nominal Independent Variable
   - One Nominal Blocking Variable
   - One Continuous Dependent Variable

Randomized Block F-Test Assumptions

1. Normality
   - Probability Distribution of each Block-Treatment combination is Normal

2. Homogeneity of Variance
   - Probability Distributions of all Block-Treatment combinations have Equal Variances

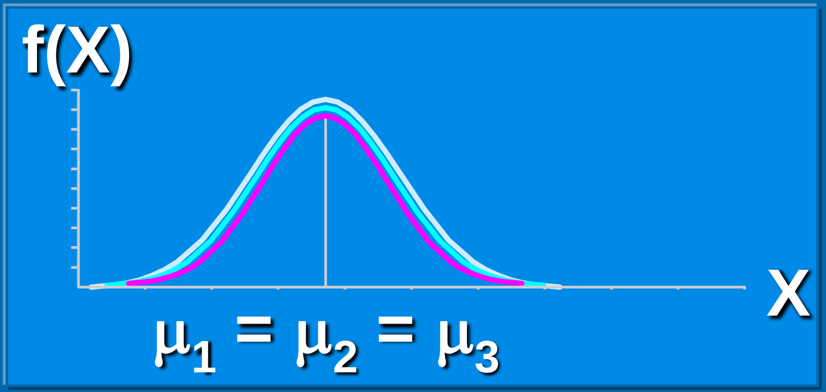
Randomized Block F-Test Hypotheses

- H_0: $\mu_1 = \mu_2 = \mu_3 = \ldots = \mu_p$
  - All Population Means are Equal
  - No Treatment Effect
- H_a: Not All μ_j Are Equal
  - At Least 1 Pop. Mean is Different
  - Treatment Effect
  - $\mu_1 \neq \mu_2 \neq \ldots \neq \mu_p$ Is **wrong**

Randomized Block F-Test Hypotheses

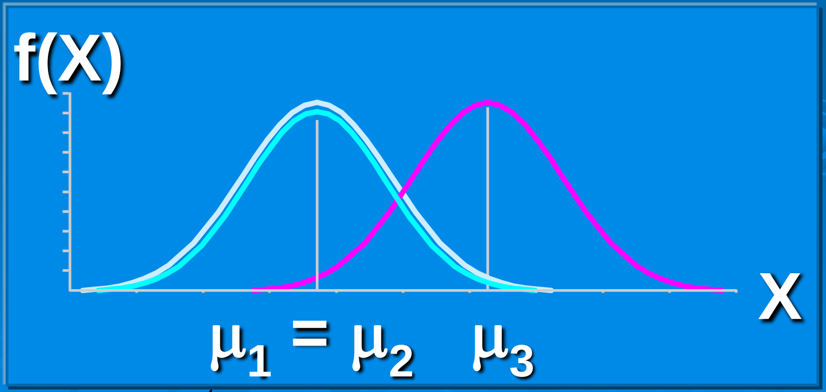
H_0: $\mu_1 = \mu_2 = \ldots = \mu_p$

- **All Population Means are Equal**
- **No Treatment Effect**



H_a: Not All μ_j Are Equal

- **At Least 1 Pop. Mean is Different**
- **Treatment Effect**
- **$\mu_1 \neq \mu_2 \neq \ldots \neq \mu_p$ Is wrong**



The F Ratio for Randomized Block Designs

- SS=SSE+SSB+SST

$$F = \frac{\text{MST}}{\text{MSE}} = \frac{SST/\left(p-1\right)}{SSE/\left(n-1-p+1-b+1\right)}$$

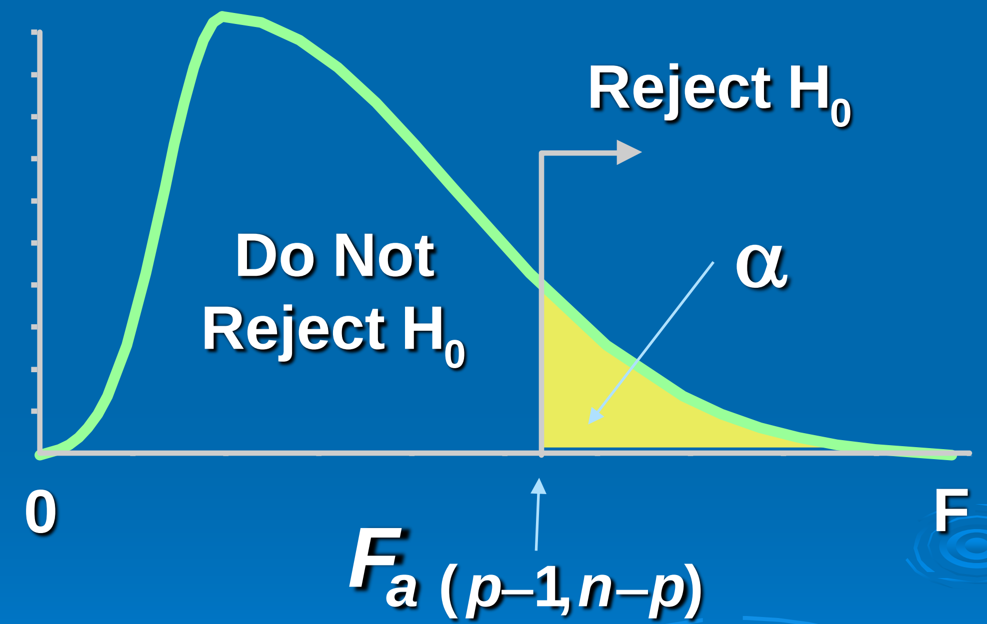
$$= \frac{SST/\left(p-1\right)}{SSE/\left(n-p-b+1\right)}$$

Randomized Block F-Test
Test Statistic

- 1. Test Statistic
  - $F = MST / MSE$
    - *MST* Is Mean Square for Treatment
    - *MSE* Is Mean Square for Error
- 2. Degrees of Freedom
  - $\nu_1 = p - 1$
  - $\nu_2 = n - b - p + 1$
    - p = # Treatments, b = # Blocks, n = Total Sample Size

Randomized Block F-Test Critical Value

If means are equal,
***F* = *MST* / *MSE* ≈ 1.**
Only reject large *F*!



Always One-Tail!

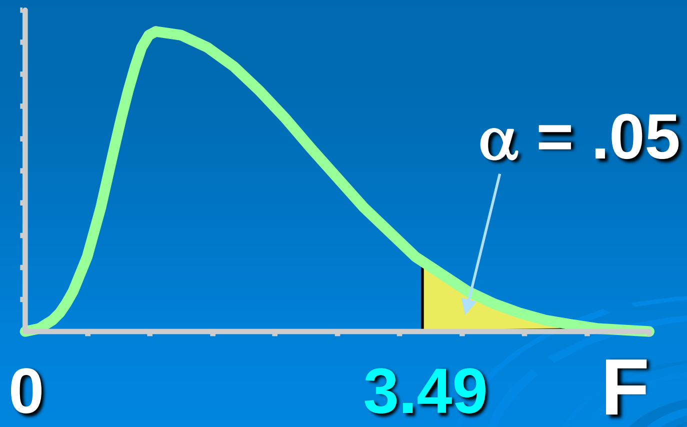
Randomized Block F-Test Example

- You wish to determine which of four brands of tires has the longest tread life. You randomly assign one of each brand (A, B, C, and D) to a tire location on each of 5 cars. At the **.05** level, is there a difference in **mean** tread life?

	Tire Location			
Block	Left Front	Right Front	Left Rear	Right Rear
Car 1	A: 42,000	C: 58,000	B: 38,000	D: 44,000
Car 2	B: 40,000	D: 48,000	A: 39,000	C: 50,000
Car 3	C: 48,000	D: 39,000	B: 36,000	A: 39,000
Car 4	A: 41,000	B: 38,000	D: 42,000	C: 43,000
Car 5	D: 51,000	A: 44,000	C: 52,000	B: 35,000

Randomized Block F-Test Solution

- H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$
- H_a: Not All Equal
- $\alpha = .05$
- $\nu_1 = 3$ $\nu_2 = 12$
- Critical Value(s):



Test Statistic:

$F = 11.9933$

Decision:

Reject at $\alpha = .05$

Conclusion:

There Is Evidence Pop. Means Are Different

Factorial Experiments

Factorial Design

- 1. Experimental Units (Subjects) Are Assigned Randomly to Treatments
  - Subjects are Assumed Homogeneous
- 2. Two or More **Factors** or Independent Variables
  - Each Has 2 or More Treatments (Levels)
- 3. Analyzed by Two-Way ANOVA

Advantages of Factorial Designs

1. Saves Time & Effort
   - e.g., Could Use Separate Completely Randomized Designs for Each Variable
2. Controls Confounding Effects by Putting Other Variables into Model
3. Can Explore Interaction Between Variables

Two-Way ANOVA

1. Tests the Equality of 2 or More Population Means When Several Independent Variables Are Used

2. Same Results as Separate One-Way ANOVA on Each Variable
   - But Interaction Can Be Tested

Two-Way ANOVA Assumptions

1. Normality
   - Populations are Normally Distributed
2. Homogeneity of Variance
   - Populations have Equal Variances
3. Independence of Errors
   - Independent Random Samples are Drawn

Two-Way ANOVA Data Table

Factor A	Factor B			
	1	2	...	b
1	Y_{111}	Y_{121}	...	Y_{1b1}
	Y_{112}	Y_{122}	...	Y_{1b2}
2	Y_{211}	Y_{221}	...	Y_{2b1}
	Y_{212}	Y_{222}	...	YX_{2b2}
:	:	:	:	:
a	Y_{a11}	Y_{a21}	...	Y_{ab1}
	Y_{a12}	Y_{a22}	...	Y_{ab2}

Observation k

Y_{ijk}

Level i Factor A

Level j Factor B

Two-Way ANOVA
Null Hypotheses

1. No Difference in Means Due to Factor A
   - $H_0: \mu_{1.} = \mu_{2.} = \ldots = \mu_{a.}$
2. No Difference in Means Due to Factor B
   - $H_0: \mu_{.1} = \mu_{.2} = \ldots = \mu_{.b}$
3. No Interaction of Factors A & B
   - $H_0: AB_{ij} = 0$

Two-Way ANOVA
Total Variation Partitioning

Total Variation

SS(Total)

Variation Due to Treatment A

- SSA

Variation Due to Treatment B

SSB

Variation Due to Interaction

SS(AB)

Variation Due to Random Sampling

- SSE

Two-Way ANOVA Summary Table

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F
A (Row)	a - 1	SS(A)	MS(A)	MS(A) / MSE
B (Column)	b - 1	SS(B)	MS(B)	MS(B) / MSE
AB (Interaction)	(a-1)(b-1)	SS(AB)	MS(AB)	MS(AB) / MSE
Error	n - ab	SSE	MSE	
Total	n - 1	SS(Total)		

Same as Other Designs

Interaction

1. Occurs When Effects of One Factor Vary According to Levels of Other Factor

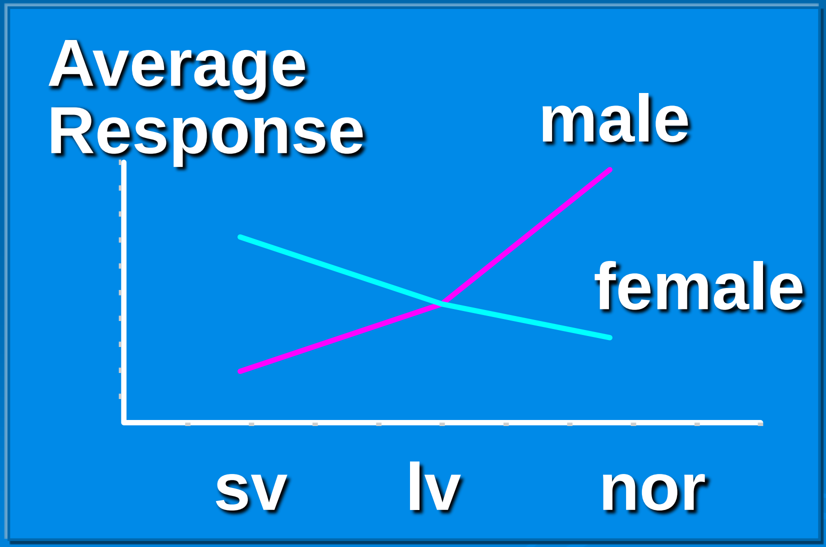
2. When Significant, Interpretation of Main Effects (A & B) Is Complicated

3. Can Be Detected
   - In Data Table, Pattern of Cell Means in One Row Differs From Another Row
   - In Graph of Cell Means, Lines Cross

Graphs of Interaction

Effects of Gender (male or female) & dietary group (sv, lv, nor) on systolic blood pressure

Interaction



No Interaction